

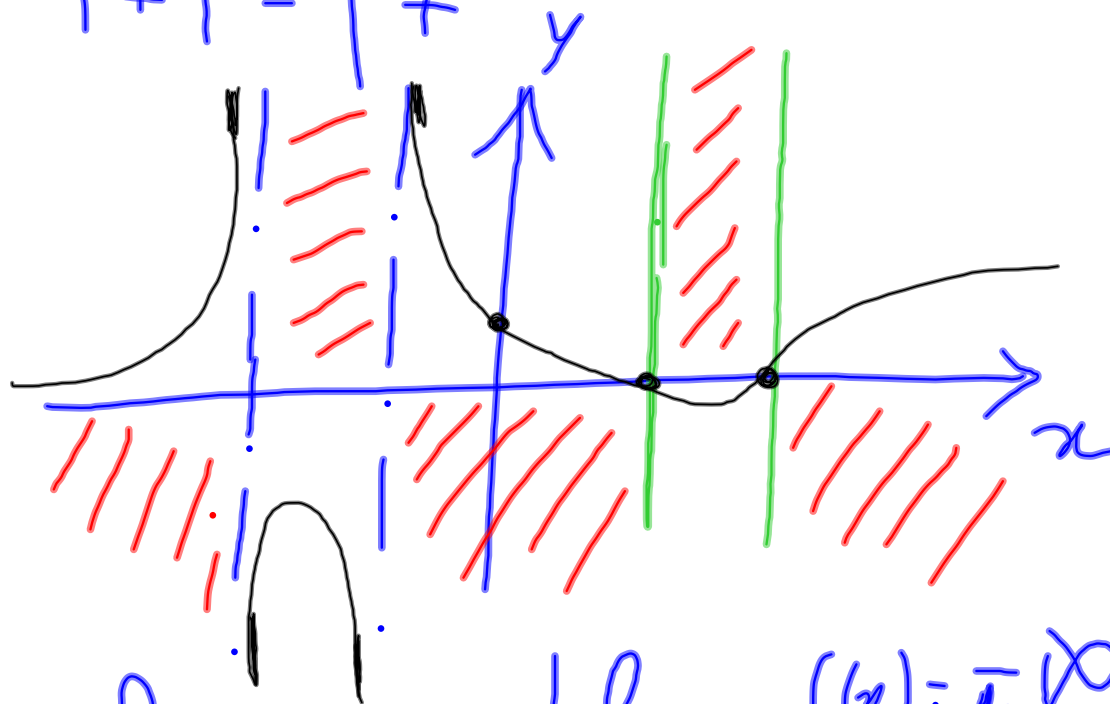
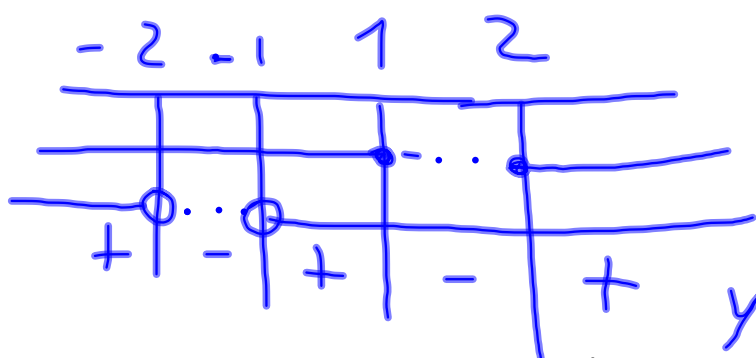
$$f(x) = \frac{x^2 - 3x + 2}{x^2 + 3x + 2}$$

$$D = \mathbb{R} - \{-1, -2\}$$

$$A(1, 0)$$

$$B(2, 0)$$

$$C(0, 1)$$



$$\lim_{x \rightarrow -2^-} f(x) = +\infty$$

$$\lim_{x \rightarrow -2^+} f(x) = -\infty$$

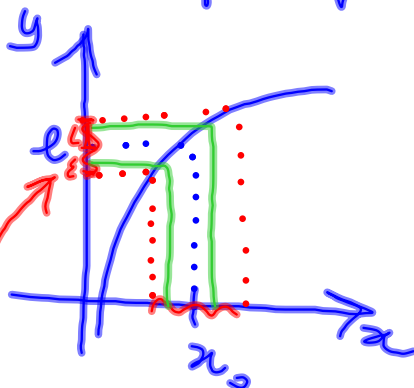
$x = -2$ A.V.

$$\lim_{x \rightarrow -1^-} f(x) = +\infty$$

$$\lim_{x \rightarrow -1^+} f(x) = -\infty$$

$x = -1$ A.V.

Limite finito per x che tende a un valore finito



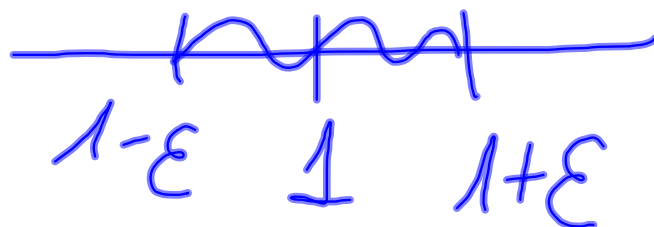
$$\lim_{x \rightarrow x_0} f(x) = l$$

$$\lim_{x \rightarrow 1} (x+2) = 3$$

$$\forall \epsilon > 0 \exists I(1) : \forall x \in I(1) - \{1\} \\ |f(x) - 3| < \epsilon$$

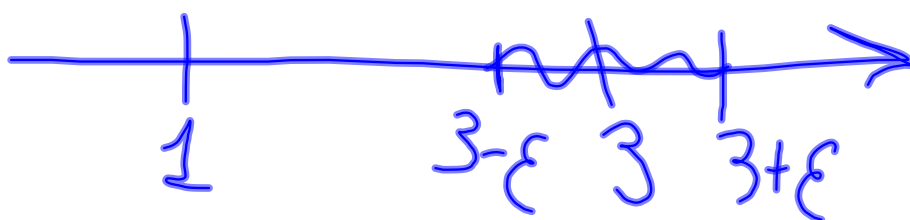
$$|x+2-3| < \epsilon ; |x-1| < \epsilon$$

$$- \epsilon < x-1 < \epsilon ; 1-\epsilon < x < 1+\epsilon$$

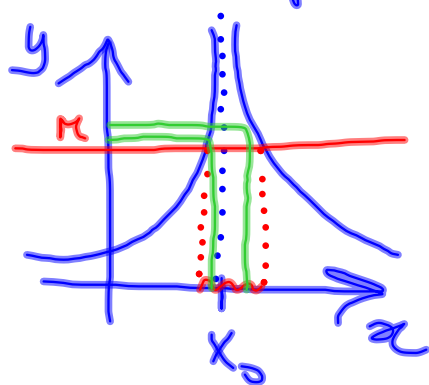


$\lim_{x \rightarrow 1} (x+2) = 5$ non è verificato.

$$|x+2-5| < \varepsilon; \quad |x-3| < \varepsilon;$$
$$-\varepsilon < x-3 < \varepsilon; \quad 3-\varepsilon < x < 3+\varepsilon$$



Limite infinito per x che tende a un valore finito



$$\lim_{x \rightarrow x_0} f(x) = +\infty$$

$$\forall M > 0 \exists I(x_0) :$$

$$\forall x \in I(x_0) - \{x_0\}$$

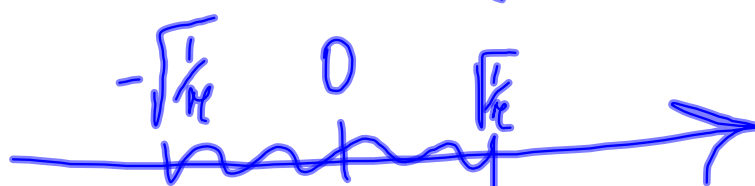
$$f(x) > M$$

$$\underline{\text{Es:}} \quad \lim_{x \rightarrow 0} \frac{1}{x^2} = +\infty$$

$$\forall M > 0 \exists I(0) : \forall x \in I(0) - \{0\}$$

$$\frac{1}{x^2} > M ; \quad x^2 < \frac{1}{M}$$

$$-\sqrt{\frac{1}{M}} < x < \sqrt{\frac{1}{M}}$$



$$\begin{array}{l} \text{oss.} \\ 3 < 4 ; \frac{1}{3} > \frac{1}{4} \end{array}$$

Asintoto verticale

Una funzione $y = f(x)$ ammette la retta $x = x_0$ come asintoto verticale se:

$$\lim_{x \rightarrow x_0} f(x) = \infty$$

$$f(x) = \frac{1}{x}$$

$$\frac{1}{0} = \infty$$

$$x \neq 0 \quad D = \mathbb{R} - \{0\}$$

$$\frac{1}{1} > \frac{1}{2} > \frac{1}{3} > \frac{1}{4} > \dots \longrightarrow 0$$

$$\frac{1}{\frac{1}{n}} = n \quad \lim_{x \rightarrow 0} \frac{1}{x} = \infty$$

Disegnare il grafico probabile della funzione:

$$f(x) = \frac{x^2 - 4}{x^2 - 9}$$

$$y = \frac{x^2 - 4}{x^2 - 9}$$

$$\frac{x^2 y + 9y}{x^2 - 9} = \frac{x^2 - 4}{x^2 - 9}$$

$$\underbrace{x^2}_3 \underbrace{y}_1 - \underbrace{9y}_1 - \underbrace{x^2}_2 + \underbrace{4}_0 = 0$$

$$F(x, y) = 0$$

$$f(x) = \frac{x^3 - x^2}{x^4 - 16}$$

$$f(x) = \frac{x^3 - x^2}{x^4 - 16}$$

1) fun. alg. raz. fr. di 5° grado

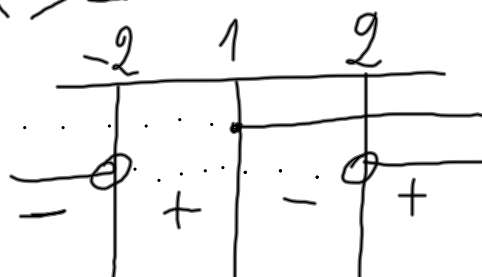
2) $x^4 - 16 \neq 0; (x^2 - 4)(x^2 + 4) \neq 0$

$x^2 \neq 4; x \neq \pm 2$

$D = \mathbb{R} - \{-2, 2\}$

3) $f(-x) = \frac{-x^3 - x^2}{x^4 - 16} \neq \pm f(x)$ non ammette simmetrie

4) $N \geq 0; x^2(x-1) \geq 0 \quad x \geq 1$
 $D > 0; x < -2 \vee x > 2$



5) $\lim_{x \rightarrow 2^+} f(x) = +\infty$
 A(1, 0)
 B(0, 0)
 $x = -2$ A.V.

$\lim_{x \rightarrow 2^-} f(x) = +\infty$
 $x = 2$ A.V.

$\mathbb{R} - \{-2, 2\}$
 $x^2 + 4 \neq 0$

